Comparing High-Frequency Measurements of Complex-Valued Network Parameters

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Introduction

This paper describes a simple practice to assess differences between measurements of electrical network parameters acquired using two different high-frequency instruments. It's an enhancement over qualitative comparisons where we just plot our own measurements along with results collected from a reference system. The method stops short of applying a more rigorous and appropriate multivariate comparison method based on student T-tests. Our entry-level method is based on NIST work [1-5] and is offered first to help us appreciate the concept of differences in complex quantities, and second to make a first-pass assessment of the differences we observe between two measurement systems.

We describe our method in the context of scattering parameters **S**, but it would work equally well for other complex-valued network parameters like impedance **Z**. In writing this paper, we are thinking about measurements made with either a frequency-domain vector network analyzer (VNA) or a high-speed time-domain reflection and transmission (TDR/T) oscilloscope that computes network parameters as functions of frequency. We are also thinking about the specific problem of gaining confidence in our test system by comparing our measurements of network parameters to those made by a reference lab on a shared set of verification devices.

In short, the method finds differences in *S*-parameters measured on two systems over a set of verification devices. The magnitude of a specific S_{ij} difference is averaged over the ensemble of verification artifacts, and the mean difference is taken as an estimate of the system differences for that particular S_{ij}. If the mean ensemble difference is greater than an estimate of the instruments' combined uncertainty, we say that that the two systems do not agree in their measurements of that S_{ij}. There could be many causes why they do not agree, but we do not examine the causes in this paper.

Network Analyzer Errors

Assuming that our measurement system is linear and time-invariant (though we should certainly test the assumption at some point), we may describe our test set-up with a calibration error model such as the idealized 12-term model commonly employed today in two-port vector network analyzers [6-7].

However, since no system model nor calibration standards are perfect, and our measured signals will include additive electrical noise, and our operator will not be able to repeat an electrical connection to a standard or device under test (DUT), we will make observations that are uncertain to some degree.

What is the uncertainty due to repeatability limits?

We can easily obtain estimates of measurement uncertainty due to noise and repeatability issues by making a sufficient number of observations on a system.

We calibrate the measurement system as usual (instead of calibrating *per se*, you may be normalizing TDR/T data using the characteristic impedance of a precision coaxial line as the reference), then we acquire repeated *S*-parameter measurements from representative devices in our verification kit. The observations include multiple electrical measurements and multiple device connections. If we were working with a two-port analyzer, we would make at least 9 total observations of the S_{ij} to estimate our repeatability.

For each S_{ij} at each frequency point, we simply estimate the standard deviation $\sigma(f)$ for the real part and imaginary parts independently over the set of all observations. We take the average of the real and imaginary σ to describe the variability in the complex S plane with a single interval $\sigma_{ij}(f)$ for each $S_{ij}(f)$ (Fig. 1). Doubling the value of σ_{ij} would approximate the 95% confidence interval u_{ij} in S_{ij}.

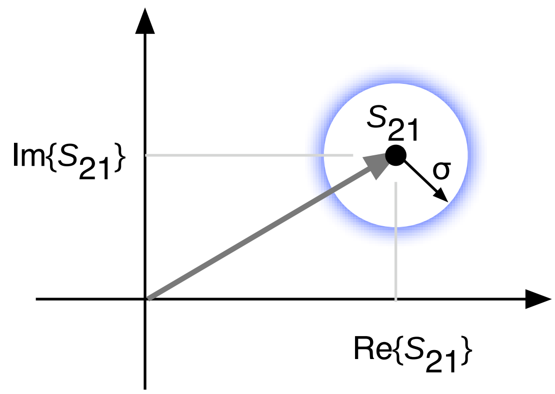

Fig.1. Depiction of 1-σ repeatability bound about a complex-valued S-parameter at one frequency, $S_{21}(f)$.

We should state again that we know and use more rigorous approaches [8], and that we are only attempting to introduce here the concepts and to provide a quick sanity check method.

What is the uncertainty due to system model and calibration standards errors?

In our tests for agreement between two measurement systems, we do not want to take the time to estimate the uncertainty due to improper system model or induced by calibration standards since it's a bit involved. Instead, our method is going to test whether there is a difference between two systems that is bigger than the repeatability uncertainty.

Since calibration errors are likely to be the main source of discrepancy in observations made on two distinct systems, we could state that our method is really testing whether two systems share the same systematic errors or not.

Network Parameter Measurement Comparison

To simplify our description, let's consider a comparison at one frequency point. Since network parameters are functions of frequency, we would need to make sure that both systems will measure at the same exact frequency values. If so, the method can be applied at each frequency point independently, so it can show changes in agreement over frequency. For now, just consider a single frequency point reported by both systems.

The method starts by calibrating both measurement systems. Both calibration methods might be different, based on what we are intending to test. If you are testing your results against another lab that makes measurements the same way as you, it's best if both labs perform identical calibration or normalization.

The method continues by building a repeatability table for both measurement systems. When estimating the repeatability bounds σ_{ij}, it's possible the σ_{ij} will vary based on the value of S_{ij}. In our method, the tables hold the largest σ_{ij} found in the repeatability experiments. We would generate a table of σ_{ij} at each frequency point independently.

We then acquire multiple measurements of the corrected *S*-parameters from our set of verification devices, and compute the complex differences between System A measurements and System B measurements. The difference between any two measurements *m* of a specified *S*-parameter S_{ij} for a specific device *n* is a vector with length:

$$\delta_{ij,mn} = \left| S^{A}_{ij,mn} - S^{B}_{ij,mn} \right|,$$

as illustrated in Fig. 2.

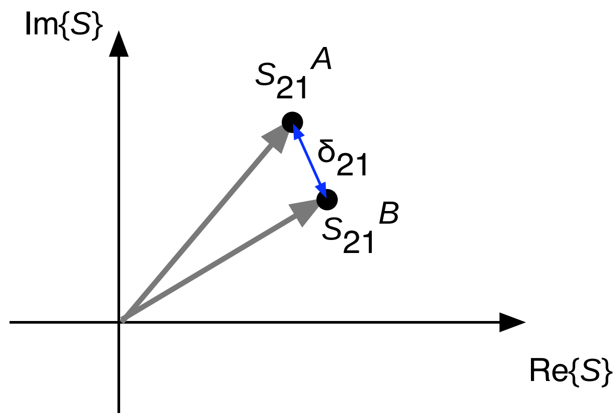

Fig. 2. Difference in two calibrated S_{21} values measured with System A and System B. The difference δ is the magnitude of the vector difference between this single pair of complex values.

All the differences in S_{ij} between the System A and System B measurement sets are summarized using the mean difference over the ensemble of measurements:

$$\Delta_{ij} = \frac{1}{NM} \sum_{n=1}^{N} \sum_{m=1}^{M} \delta_{ij,mn}.$$

This gives an estimate of the mean deviation between a specific *S*-parameter measured on the two systems.

With the typical measurement difference Δ_{ij} and our repeatability estimates for both systems σ_{ij}^{A} and σ_{ij}^{B}, the method tests whether the difference is bigger than the sum of the repeatability bounds. We say System B does not agree with System A when Δ_{ij} does not fall within the repeatability limits of the two systems: $\Delta_{ij} \geq \sigma_{ij}^{A} + \sigma_{ij}^{B}$. In other words, the differences in systematic errors between the two systems are likely to be larger than the uncertainty due to repeatability sources (Fig. 3a).

If $\Delta_{ij} \leq \sigma_{ij}^{A} + \sigma_{ij}^{B}$, we say the differences in systematic errors are not significantly larger than the predicted repeatability error. In other words, the systems appear to agree within the repeatability limits (Fig.3b).

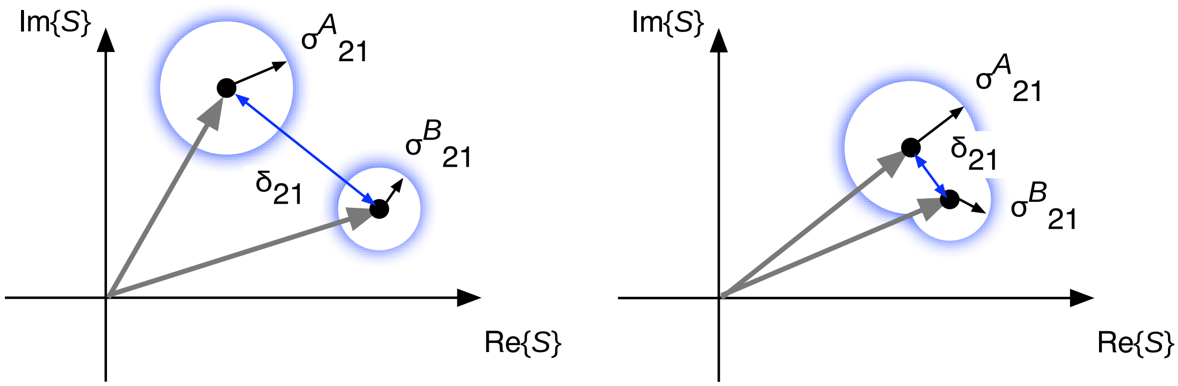

Fig. 3. Summary of differences in S_{21} measurements between System A and System B showing **a** (left), Δ_{21} falling outside the estimated repeatability bounds $\sigma_{21}^{A} + \sigma_{21}^{B}$; and **b** (right), Δ_{21} falling inside $\sigma_{21}^{A} + \sigma_{21}^{B}$.

Example System Comparison

This section provides an example network analyzer comparison, illustrating the application of our method to two dissimilar systems used to acquire *S*-parameters: an experimental time-domain network analyzer (TDNA) system based on TDR/T and a commercial frequency-domain network analyzer (FDNA).

We performed a multiline TRL [9-10] calibration on both systems using known transmission line standards with probe contacts. Our set of verification devices consisted of passive devices and transmission lines with the same probe contacts, but were intentionally chosen to be different than the structures used as the calibration standards. Our measurements in this comparison covered a frequency span of 50 MHz to 12 GHz, which is the limit over which the two systems overlap.

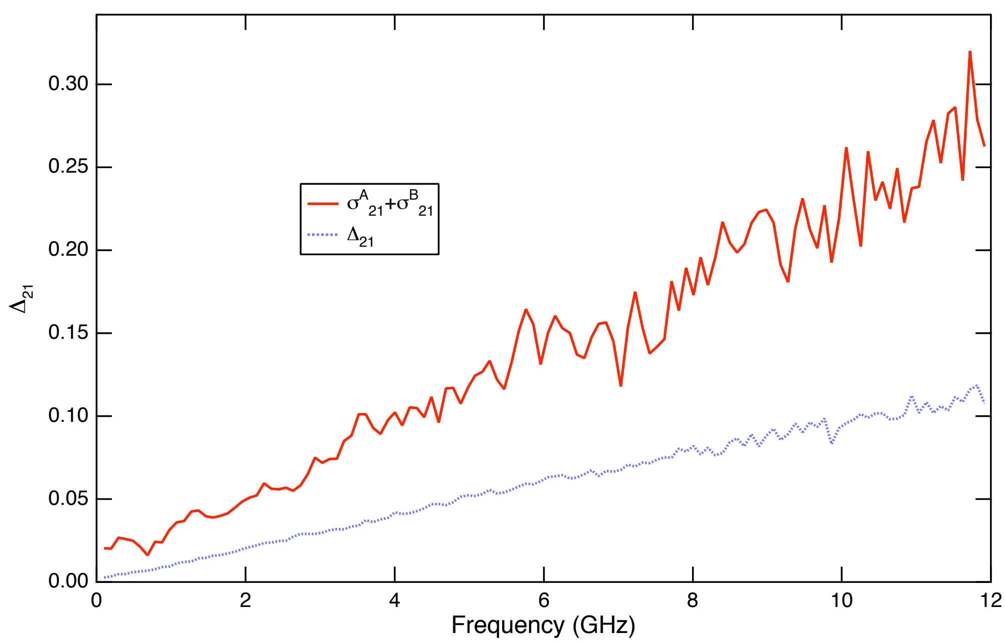

Fig. 4. Comparison of Δ_{21}, the mean difference in S_{21}, to the sum or repeatability estimates for a TDNA and an FDNA network measurement system.

Figure 4 plots the sum of the repeatability bounds ($\sigma_{21}^{A} + \sigma_{21}^{B}$) in S_{21} along with the mean difference Δ_{21} determined using multiple measurements the devices in the verification kit. Looking at the plots, we would say we are not detecting significant differences between the two systems over the entire frequency band.

Of course both systems could be making large mistakes. Since we are not testing for agreement, we are really saying that we cannot find a difference between the systematic errors that is larger than the “random” errors.

Summary

We have described a simple method of comparing two distinct vector network analyzer systems (VNA or TDR/T) using measurements of a specific set of verification devices. At each frequency point in our sweeps, this method produces a single scalar estimate of the average differences between an ensemble of measurements made with two test set-ups. The resulting average difference can be used as an estimate of S_{ij} differences between two systems. When compared to the estimated repeatability bounds, it can also be used to make a first-pass assessment about the agreement, or lack of agreement, between the two systems. This is useful to engineers who are establishing a new measurement capabilities, and to experienced users of VNAs who routinely check the consistency of their techniques.

Though more rigorous methods exist, this entry level method clarifies the basic concepts and serves as a quick test when time is not available to perform complete uncertainty analyses.

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